FROM ZERO TO INFINITY: INDIA’S RICH HERITAGE

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INTRODUCTION

While the contributions of the West in the realms of Mathematics and Science are well-known and well-documented, unfortunately that is not the case when it comes to significant contributions that have originated in the East, notably India and China. This article is an attempt at presenting a balanced perspective and creating an awareness of India’s role in molding civilization, as we know it today. This awareness is important not only for Indians, but also for westerners who have been programmed to think of India only as a ‘Third World’ country, often with their exposure to India limited to movies like ‘The Slum Dog Millionaire’.

When my daughter, Tara, was an undergraduate student at Temple University she had to take a mandatory course entitled ‘Intellectual Heritage’. She found the course interesting, but very restrictive in that the entire course was devoted to western civilization, with no hint of any civilization in the East. Tara brought this huge gap in the course to the attention of University personnel, who later revised the course content. Hopefully, more American students are now getting exposed to intellectual heritage with an eastern flavor as well. It is hoped that this article will further narrow the intellectual gap that has existed in America for a long time.

I was always aware of the profundity of concepts such as ‘zero’ and ‘infinity’ that have emanated from India, but had not realized the extent and depth of India’s contributions in diverse fields, until my recent almost epiphanic experience in Cambodia led me to further explore my own roots. I was overcome with a bizarre sense of ‘déjà vu’ when I first laid eyes on the Angkor Wat temple in Cambodia. I was clearly inspired by the uncanny resemblance between Angkor Wat and the traditional South Indian temples I grew up with in Kerala and in Karnataka. A tourist from New Zealand asked me if I had hired a guide to explain the intricacies of Angkor Wat, and my response was that I did not need a guide because of my familiarity with South Indian temples, which are amazingly similar in architecture to Angkor Wat. I said that with a sense of pride in my heritage which is epitomized in Angkor Wat, a beautiful monument proclaimed by the UN as a world heritage site.

Angkor Wat, an architectural marvel built in the 12th century, was primarily based on the Dravidian architecture, which emerged thousands of years ago in South India. It consists of pyramid shaped temples with intricate carved stone and a step design with many statues of deities, warriors, kings, and dancers. This type of architecture is mentioned as one of three styles of temple building in the ancient Indian book ‘Vastu Shastra’. (Santhanam, 2018)

While traveling in Cambodia and trying to decipher the writing on sign boards, I was intrigued by the similarity between the Cambodian script and the Kannada script used in the state of Karnataka in South India. This similarity can, of course, be traced to a common source, and in this case, it is the Pallava script, which was developed in South India during the 3rd century. Cambodia’s link to India is further manifest in the name ‘Angkor Wat’, which means city temple. ‘Angkor’ is the vernacular version of the original word ‘nokor’, derived from the Sanskrit word ‘nagara’, which means city and ‘Wat’ is the vernacular version of the Sanskrit word ‘vata’, which means temple (Hebbar, 2018).

The question naturally arises: How did Indian culture and architecture have such a profound impact on Cambodia? The answer lies in the cultural and commercial interactions that took place between South India and Cambodia, dating back to a few centuries before Christ. During
those times there was a major maritime highway linking India, China, South East Asia, Sri Lanka, Africa, and Rome.

There was abundant South Indian influence on Cambodian art and culture during the rule of the Pallavas (3rd to 8th AD) and the Cholas (9th to 13th AD) in South India. The rulers of Cambodia during this time had a suffix of ‘Varman’ to their names, in a way similar to the Pallava kings of Kanchipuram. While the Cholas of Tanjavur eventually defeated the Pallava Varmans in the 8th century, the Khmer kingdom in Cambodia flourished well into the 14th century. Artisans from the Chola Kingdom helped construct Angkor Wat. It should probably come as no surprise that the chief priest appointed by Suryavarman II, the builder of Angkor Wat, was an Indian scholar (Damodar Pandita) who provided the guidelines for building the temple. (Kasturi and Suresh, 2012).

While India’s contributions in the field of architecture are impressive, they represent only the tip of the iceberg when you look at the entire spectrum of contributions India has made in diverse fields such as Mathematics, Physics, Chemistry, Geography, Astronomy, Engineering, Medicine and Surgery, Music, Arts, and of course, Philosophy. This article focuses on India’s contributions in the field of Mathematics. There are several scholarly treatises dealing with this topic. The purpose of this article is to make the contents of these treatises less arcane and to present them in a comprehensive and comprehensible manner from a layman’s perspective.

**ZERO**

It is difficult to imagine a world without zero, although paradoxically, ‘zero’, which was conceived in India, was originally named ‘shunya’, the Sanskrit (an ancient Indian language) word for ‘void’ or ‘nothing’. Upon further reflection, however, the appropriateness of this nomenclature becomes evident when one considers an indicator variable (e.g. in regression models used in statistics) for indicating the presence or absence of an attribute (e.g. smoker vs non-smoker), with 1 denoting ‘presence’ and 0 denoting absence.

Without zero, there would be no decimal system for counting and keeping track of our financial transactions. The entire banking system would go haywire without zeros inserted in the proper places to represent numbers. The numbers ten, one hundred, and one thousand are vastly different numbers, and yet they can all be displayed easily with the digit ‘one’ followed by the appropriate number of zeros. The value of any digit depends on its position in the whole number, and zero functions as a placeholder. A zero inadvertently omitted, misplaced, or added, can create havoc.

Aside from providing a simple and elegant way to represent numbers, the decimal system, with zero at its core, made arithmetic operations of addition, subtraction, multiplication, and division far easier than they were with number systems that were being used before the advent of the decimal system. In sharp contrast, the Roman number system, which was in wide use in the West until the thirteenth century, was a cumbersome system, with Latin letters denoting numbers (I for 1, X for 10, L for 50, C for 100, M for 1000) and no zero. For instance, the number, 4370 had to be written as MMMMCCCLXX. Multiplying this number by XVIII must have been a challenging exercise. Clearly, the Roman number system, which is not easily amenable to arithmetic operations, could only have impeded any potential development of mathematical concepts in the west.
The Babylonian system, which preceded the Roman system by about two millennia, was an inefficient system with base 60 and no clear-cut place-holding zero, again lacking the versatility of the decimal system with zero included as one of the ten numerals. The Mayan numerals, which go back to 37 BC, used base 20 and 3 basic symbols including zero which was meant to be a placeholder. But the Mayan system lacked economy of notation, requiring more than one symbol and position to represent the numbers 6 to 19, and as a result zero was not a perfect positional element as it is in our decimal system. Georges Ifrah called the Mayan numbers a failed system - it was not one that survived the test of time. In contrast, the use of zero as a placeholder, together with the use of one unique symbol to represent each of the other nine numerals, has contributed to the effectiveness of the decimal notation. (Aczel, 2015; Ifra, 2000; Dutta, 2015)

Zero also plays a key role in the binary system used in computer technology, whereby data is represented in bits, each with a value of either 1 or 0, encoded by a switch that is either ‘on’ or ‘off’. Any number can be represented as a sequence of bits. Decimal numbers use base 10, while binary numbers use base 2. Decimal numbers and binary numbers are governed by similar rules. Every time a number moves to the left, the value increases by a power of ten in the decimal system and by a power of two in the binary system.

The invention of zero has led to many other interesting mathematical concepts, including the concept of negative numbers, which are used extensively in Mathematics, Science, and Business. Graphical displays, commonly used in all branches of Science and in Business, are based on the Cartesian system, with (0,0) as the origin, and axes marked with negative as well as positive numbers. The extensive use of graphics in today’s business and scientific scenarios, for handling vast amounts of data efficiently, underscores the importance of zero. (Kline, 1985)

Zero is featured prominently in Calculus, a branch of Mathematics dealing with limits of functions, derivatives (i.e. rates of change) and integrals (i.e. areas and volumes). Calculus was developed out of a need to understand continuously changing quantities. Newton, one of the founders of calculus, was trying to understand the effect of gravity which causes falling objects to constantly accelerate. Newton was trying to determine the speed of a falling object when it strikes the ground. If an object is traveling with constant speed, this speed can be derived easily by calculating the distance traveled between any two points in time and dividing this distance by the difference between these time points. However, this simple method cannot be applied to Newton’s scenario involving an object falling with increasing speed over time. This scenario requires the calculation of the instantaneous speed attained by the falling object at a specific point in time, and this is where zero and the concept of ‘derivative’ come in. Instantaneous speed at a specific time point can be derived approximately by iteratively calculating the speed over successively smaller time intervals just preceding the specific time point. More rigorously, instantaneous speed can be defined as the limit of the speed over a specific time interval as the interval approaches zero. This is precisely the derivative, at that time point, of the trajectory of the falling body.

Calculus, which deals with limits of numbers as they approach zero, has revolutionized the scientific approach, leading to significant advances in physics, engineering, economics, statistics, and finance. Basically, calculus provides a framework for modeling and controlling systems based on the effects of changing conditions on these systems.
The decimal system was already in practical use during the Indus Valley civilization which flourished in India from 3000 BC to 1500 BC. It is not known exactly when the use of zero as a placeholder appeared in the context of the decimal system. In his prosody text, “Chandah-sūtra”, Piṅgalā (c. 300 BC), a prominent Indian mathematician and musical theorist, gives rules to associate a unique number with each meter and to recover from a number the meter which it represents; they are essentially formulae for converting a binary representation of a number into its decimal representation and vice versa. Further, Piṅgala’s algorithms involve the use of dvi (two) and shūnya (zero) as distinct labels. Thus, there is every indication that a decimal place value notation with zero existed at the time Piṅgala was formulating his rules. As pointed out by Datta (1929), the concept of decimal place (sthāna) is explicitly mentioned in the Jaina text, Anuyogadvāra-sūtra (c. 100 BC). Here, the total number of human beings is described variously as the product of $2^{64}$ and $2^{32}$ (i.e., $2^{96}$), as a number which can be divided by two 96 times and also as a number, which, in terms of denominations like koṭi-koti (10$^{14}$), occupies 29 places (sthāna). Indeed, the number $2^{96}$ (79228162514264337593543950336) requires precisely 29 digits in the decimal notation, with 0 occurring in the thousand’s place in this 29-place decimal expansion. Thus, the reference to 29 places for this number indicates the prevalence, in some form, of the concept of zero as a placeholder as early as 100 BC. (Pearce, 2002; Dutta, 2015; Datta, 1929)

Brahmagupta, the great 7th century Indian Mathematician/Astronomer, made the brilliant conceptual leap to include zero as a number in its own right, rather than merely as a placeholder, a blank or empty space within a number, as it had been done until that time. In his epoch-making treatise, ‘Brahmasphutasiddhanta’, written in 628 AD, Brahmagupta (598 AD-668 AD) introduced the seminal concept of zero as a number, which is so basic to all of mathematics. The introduction of zero as a number which could be used in calculations and mathematical equations, has indeed revolutionized mathematics. Brahmagupta, who was born in the ancient Indian city of Ujjain, a major center for astronomy and mathematics, was the first to establish basic mathematical rules for dealing with zero: $1 + 0 = 1$; $1 - 0 = 1$; and $1 \times 0 = 0$. Brahmagupta’s view of numbers as abstract entities, rather than those used just for counting, allowed him to make yet another conceptual leap, with profound consequences for the future direction of mathematics. Previously, the result of subtracting 2 from 1, for example, was considered meaningless. However, Brahmagupta, could perceive it as a negative number, which he referred to as “debt” as opposed to “property”, which he used for designating a positive number. He went on to formulate rules for dealing with negative numbers (e.g. a negative number times a negative number is a positive number, a negative number times a positive number is a negative number, etc.). He further pointed out that quadratic equations in theory have two possible solutions, one of which could be negative. He used the initials of the names of colors to represent unknowns in his equations in a manner akin to modern algebraic notation. He also demonstrated the use of mathematics and algebra in predicting astronomical events. As a fitting tribute to his genius, Brahmagupta was referred to as ‘Ganita Chakra Chudamani’ (a Sanskrit word for ‘the gem in the circle of mathematicians’) by Bhaskara II, who was a great mathematician in his own right. (Mastin, 2010; The Columbia Encyclopedia, 2000 (Encyclopedia.com); O’Connor and Robertson, 2002; Joseph, 2011; Colebrooke, 1817; Kaplan, 1999; Puttaswamy, 2012))

It took a few centuries before zero made its way to Europe. First, the great Arabian voyagers took the texts of Brahmagupta and his colleagues back with them, along with spices and other exotic items from India. Brahmagupta’s writings became known in the Arab world, whose ruler, King Khalif Abbasi al-Mansoor (712-775 AD), invited the Ujjain scholar, Kanaka, to lecture on Brahmagupta’s works at the new center of learning in Baghdad founded on the banks
of the Tigris. The king commissioned the translation of Brahmagupta’s writings into Arabic in 771 AD, and the translations were made by Muhammad-Al-Fazari, an astronomer in Al-Mansoor’s court. These translations had a major impact on the Arab world, eventually leading to the adoption of the Indian decimal number system by the Arabs. (Wallin, 2002; The Columbia Encyclopedia, 2000 (Encyclopedia.com); Pearce, 2002; Joseph, 2011)

Although zero reached Baghdad by 773 AD, its wide-spread use in the Arab world had to wait until the Persian mathematician, Muhammad Al-Khwarizmi (780-850 AD), who was one of the first Directors of the House of Wisdom in Baghdad, strongly advocated the Hindu number system, after recognizing its power and efficiency. The Indian numerals, 0-9, which have since become known as Hindu-Arabic numerals were soon adopted by the entire Arab world. Al-Khwarizmi wrote a text called ‘al-Jam wal-tafriq bi hisab-al-Hind’ (Addition and Subtraction in Indian Arithmetic), which was later translated into Latin as ‘Algoritmi de numero indorum’. Through this text, the decimal number system and Brahmagupta’s algorithms for arithmetic have spread throughout the world. Al-Khwarizmi called zero ‘sifr’, which means ‘nothing’ in Arabic. Thanks to the conquest of Spain by the Moors, zero finally reached Europe; by the middle of the twelfth century, translations of Al-Khwarizmi’s work reached the English shores. (Wallin, 2002; Mastin, 2010)

While his colleagues at the House of Wisdom pored over Greek and Byzantine texts, Al-Khwarizmi was drawn toward writings from India. He had the librarians trawl their archives for papers brought from India in an earlier era. After much searching, a treasure trove of Indian texts, including Brahmagupta’s ‘Brahmasphutasiddhanta’ was uncovered. Using bilingual aides, Al-Khwarizmi began to render the foreign characters into Arabic. And with each day, he discovered new ideas and symbols that initially mystified him, but gradually became clear. He was fascinated by the Indian system of symbols for representing the quantities 1 to 10 and combinations of these symbols for representing ever increasing quantities rising to infinity. He could see that using this system of symbols was an infinitely better way of representing numbers, compared to the methods that were being used then by the Arabs and others in the Mediterranean-Mesopotamian region. (Qatar Foundation for Education, 2009)

Al-Khwarizmi was especially intrigued by the symbol, a black dot, used by Brahmagupta to represent zero. When Al-Khwarizmi asked his translators for an explanation of this symbol, he was told that it meant nothing. Thinking it was a joke, Al-Khwarizmi pressed for an explanation, and was finally told that the black dot was used for representing the quantity of nothing. Al-Khwarizmi was baffled by the meaning of zero and the mystery surrounding it. Brahmagupta’s text also revealed a whole system of negative numbers, less than zero, stretching as far into the negative side of infinity as those greater than zero stretching into the positive side of infinity. Al-Khwarizmi quickly adopted all these foreign concepts and symbols and disseminated them far and wide through his own writings. He pointed out the Indian origin of the number system he was using in his work; however, subsequent translations of his work attributed all the work and the Indian numerals to Al-Khwarizmi. Clearly, that was not Al-Khwarizmi’s intent; his only intent was to present and preserve Brahmagupta’s concepts in a document that could be used by the Arabs. Al-Khwarizmi’s Arabic version embodying Brahmagupta’s concepts, called ‘Sindhind’, was unveiled in 825 AD. (Qatar Foundation for Education, 2009; Joseph, 2011)
The Italian mathematician, Fibonacci (of the famous Fibonacci sequence) was instrumental in bringing the Hindu-Arabic numerals to Europe through his book, ‘Liber Abaci’, or ‘Abacus book’, published in 1202. This book described the 9 Indian digits 1 to 9, and a symbol, 0, which Fibonacci called zephirum, clearly a close cousin of the word ‘sifr’ (which, in Arabic, means ‘nothing’) used by Al-Khwarizmi. Fibonacci’s work quickly gained the attention of Italian merchants and German bankers, who especially appreciated the role of zero. Accountants knew their books were balanced when the positive and negative amounts of their assets and liabilities equaled zero. But governments were still suspicious of the Hindu-Arabic numerals because of the ease in which it was possible to change one symbol into another. Though outlawed, merchants continued to use zero in encrypted messages; this explains the derivation of the word ‘cipher’ meaning ‘code’ from the Arabic ‘sifr’. (Aczel, 2015; Wallin, 2002)

Indian mathematicians, and in particular, Brahmagupta, made invaluable contributions to the world of Mathematics and Science. However, they made their contributions at a time when patenting, copyrighting, and lawyers specializing in intellectual property rights did not exist. This could be why despite overwhelming evidence supporting the fact that zero and the decimal system are of Indian origin, bizarre intellectual speculations about the origin of zero never seem to cease. In his fascinating book ‘Finding Zero’, Dr. Amir Aczel acknowledges that in the book, ‘Liber Abaci’, Fibonacci clearly refers to the nine digits 1 to 9 as ‘Indian’. Dr. Aczel further states that the root of the Latin ‘zephirum’, used by Fibonacci to denote 0, has been traced to the Arabic word for zero, ‘sifr’, thus establishing a linguistic connection between the Arab zero and the new European one. (Aczel, 2015)

Dr. Aczel, who bases his judgement on Fibonacci’s work, is reasonably sure that the nine numbers 1 to 9 originated in India, but is less sure about zero and poses the question: Was zero Arabic, or Indian, or did it come from some other place? Determined to find an answer to this question, Dr. Aczel embarked on an astonishing odyssey that took him to India, Thailand, and eventually Cambodia. He found a four by four ‘magic square’ with the number 10 inscribed in it, carved into the wall of a temple in Khajuraho, a small town in India, noted for its temples adorned by statues depicting outrageously explicit sex. Although the magic square, dated 954, predated the European zeros by almost three centuries, it did not provide sufficient evidence for establishing that zero was of Indian origin. (Aczel, 2015)

Moving on with his mission, Dr. Aczel found an earlier zero in the city of Gwalior in India, where a grant of land 270 ‘hastas’ long was recorded in a temple inscription, dated 876 AD. However, from Dr. Aczel’s perspective, the Gwalior zero was also not old enough to rule out the possibility that it had come from Arabia and had gone to Arabia from Europe, since the ninth century is well within the timeframe of extensive Arab sea trade. This view was popular among western scholars in the early 20th century who were swayed by Kaye, an anti-Indian scholar, determined to spread the message that the number system including the zero numeral was either European or Arabic in origin. Dr. Aczel recognizes this claim as European bigotry, and is convinced that zero must be the creation of the Eastern mind. (Aczel, 2015; Joseph, 2011)

Following leads from various scholars, including the French archeologist, George Coedes, Dr. Aczel ended up in Cambodia where he found a zero (denoted by a dot) in an inscription on a stele, dated 683 AD. He was elated by this discovery at the end of a long journey, which at times resembled that of Indiana Jones in Steven Spielberg’s iconic film, ‘Raiders of the Lost
Dr. Aczel succeeded in getting this inscription reinstated at the Cambodian National Museum, and wrote a description to go with the display, in which he refers to an article by George Coedes, claiming that zero is an Eastern, and perhaps Cambodian invention, since this inscription predates the Arab empire, as well as the Gwalior zero, by two centuries. (Aczel, 2015; Ifra, 2000)

Dr. Aczel, along with George Coedes, managed to debunk the myth about the numerals being invented in the West. However, the suggestion that zero is perhaps a Cambodian invention is clearly far-fetched. Even Mr. Hab Touch, the Director of the Cambodian Office of Cultural Affairs, with whom Dr. Aczel met in the process of getting the 683 AD inscription restored, could not be persuaded by Dr. Aczel to accept the notion that zero could be a Cambodian invention. In fact, Mr. Touch, an expert on his country’s art and history, countered this notion by pointing out the connection with India early on in the development of the Cambodian or Khmer culture. He noted that Old Khmer, in which the inscription was written, was derived from Sanskrit, and that many of the themes depicted at Angkor Wat, such as the famous bas relief of the Churning of the Sea of Milk came directly from the Indian epics of Ramayana and Mahabharata. He went on to state that the Cambodian civilization was influenced mostly by India and that China, another powerful nation in the region, had less influence on Cambodia. Mr. Touch’s views, which are totally consistent with the facts stated earlier in the section on ‘Introduction’ about Cambodia’s link to India, suggest that the Cambodian zero, like a lot of the Cambodian culture, must have come from India.

Furthermore, Dr. Aczel refers to the ancient Bakshali manuscript written on birch bark, which was discovered in India and is presently on display in the Bodleian Library at Oxford. This document uses a symbol for zero and Sanskrit numerals to represent the digits 1 to 9. The Bakshali manuscript, which abounds with numbers expressed in decimal notation and with decimal calculations, contains a wealth of mathematical writings, including equations, ways of estimating square roots, and results involving negative numbers. According to Dr. Aczel, if this document could be dated to the second or third, or even the fourth century, it would establish that zero (and with it our entire number system) was invented very early in India. Because of the fragile nature of this document, the British authorities had not permitted radiocarbon analysis of this document, and thus, it was impossible to determine the real age of the Bakshali manuscript. (Aczel. 2015; O’Connor and Robertson, 2000; Pearce, 2002)

Carbon dating of the Bakshali manuscript has been done more recently in 2017, placing this document in the third or fourth century AD, about 500 years earlier than previously thought. The Bakshali manuscript which is now known to contain the earliest recorded zero predates the Cambodian zero by more than two centuries. (Sautoy and Ovenden, 2017; Suri, 2017)

If the more recent finding about the Bakshali manuscript had surfaced earlier, it might very well have convinced even a skeptic like Dr. Aczel that zero originated in India. Another compelling piece of evidence in support of the Indian origin of zero is, of course, the document ‘Brahmasphutasiddhanta’, written by Brahmagupta (who, coincidentally also used a dot to represent zero) in 628 AD, well before the date (683 AD) of the Cambodian inscription.

Thus, zero is clearly an Indian invention; QED. How this conclusion and the evidence supporting this conclusion eluded Dr. Aczel and others of his ilk is a bit of a mystery, perhaps rivaling the mystery of zero itself. Nonetheless, Dr. Aczel has made a significant contribution, by explaining the importance of zero in layman’s terms and pointing out interesting links
between the concepts of zero and infinity and Eastern Philosophy, which is embedded in Hinduism, Buddhism, and Jainism, the religions that originated in India.

According to Dr. Aczel, the invention of zero is perhaps the greatest intellectual achievement of the human mind, considering the fact that we live in a digital world where everything we do with a computer (or cellular phone, GPS, or anything electronic) is controlled by a string of zeros and ones. He articulates the simplicity, versatility, and efficiency of the decimal number system, in comparison to the Roman numeral system and points out that it is zero that enables the efficiency and power of the decimal system. Unlike the Roman numeral system, in which the numbers cannot be cycled, with the help of zero in the decimal system we can use the same numeral in different places to represent different numbers (e.g. 5, 50, 505, etc.). A similar, perhaps more vivid, description of the decimal system is found in Vyasas’s commentary on Patanjalis Yoga Sutras, dated around 400 AD: “In the units place, the digit has the same value, in the tens place, 10 times the value and in 100th place, 100 times the value, as a woman is called mother, daughter, and sister.” (Aczel, 2015; Vanadeep, 2012; Joseph, 2011)

It is clear that the invention of zero and the decimal system has had a profound impact on the course of human civilization, whichever way you slice it. According to Will Durant, the illustrious American author of ‘The Story of Civilization’, “It is true that even across the Himalayan barrier, India has sent to the West such gifts as grammar and logic, philosophy and fables, hypnotism and chess, and above all numerals and the decimal system. India was the mother of our race and Sanskrit the mother of Europe’s languages. She was the mother of our philosophy, mother through the Arabs, of much of our mathematics, mother through Buddha, of the ideals embodied in Christianity, mother through village communities of self-government and democracy. Mother India is in many ways the mother of us all.” (Durant, 1976)

Pierre Laplace, a famous 18th century mathematician and astronomer had this to say about the decimal system: “It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity and the great ease which it has lent to computations put our arithmetic in the first rank of useful inventions; and we shall appreciate the grandeur of the achievement the more when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity.” (Eves, 1988)

In his book ‘The Wonder that was India’, A. L. Basham, the famous Australian Indologist, wrote: “Medieval Indian mathematicians, such as Brahmagupta (seventh century), Mahavira (ninth century), and Bhaskara (twelfth century), made several discoveries which in Europe were not known until the Renaissance or later. They understood the import of positive and negative quantities, evolved sound systems of extracting square and cube roots, and could solve quadratic and certain types of indeterminate equations.” (Basham, 1954)

**INFINITY**

Dr. Aczel was convinced that there must have been something about the philosophy of the East that made the Eastern mind more amenable to completing the number system at both of its extremes: adding zero at one end and infinity at the other. Indeed, in Hindu mythology, the world was created when Vishnu, one of the three Hindu gods, woke up from his eternal slumber while reclining on the serpent, Anantha, which, in Sanskrit, means ‘endless’. This imagery
helps us visualize the underlying philosophical concept of human consciousness, the awakening of which heralds the creation of our world. The idea of infinity is expressed in the form of an endless quantity or extent, as embodied by Ananta, and in the form of an endless past: eternity until Vishnu was awakened. Unlike Hinduism which uses imagery to convey abstruse philosophical concepts, Buddhism and Jainism are devoid of such imagery and instead focus on the concepts themselves. Zero or ‘Shunya’ as it was referred to by ancient Indian mathematicians, could well be linked to the concept of ‘Shunyata’ or ‘emptiness’ in Buddhism, which is regarded as a means of attaining ‘nirvana’ or liberation. Jain thinkers understood the concept of exponentiation and realized that exponents grow extremely fast. The Jain literature deals with very high powers of 10 such as 10 to the power of 60. Thus, the three religions, Hinduism, Buddhism, and Jainism, together have given rise to the concepts of zero, infinity, and finite but extremely large numbers. (Aczel, 2015)

Bhaskara-II in his pioneering work ‘Beeja Ganitam’ gives a somewhat philosophical, yet elegant definition of infinity as follows: “A fraction having zero as the denominator has a peculiar property that can be likened to the quality of the Almighty. All the beings merge with Him during the time of dissolution (Pralaya) and they are re-created with the same grandeur and plentitude from the same Supreme Godhead. But, during these two processes, He does not undergo any changes, whatsoever. Similarly, a fraction with zero as its denominator does not undergo any change even if a very huge number is added to it or subtracted from it. They make no difference at all”. (Vanadeep, 2012)

Mathematically, infinity can be defined as the limit of n/x as x approaches 0, n being any finite number; infinity can thus be written as n/0, as alluded to by Bhaskara. Further, infinity plus or minus any finite number is still infinity, thus providing the rationale for Bhaskara’s assertions about infinity. Conversely, zero can be defined as the limit of n/x as x approaches ∞ (the symbol for infinity), n being any finite number. Thus, zero can be written as n/∞. Both zero and infinity, which are closely linked, are not physically measurable or attainable quantities, and thus they lie beyond the realm of physical reality in a manner akin to Divinity. The Universe has been characterized using the Sanskrit expression ‘Anantashunya’, meaning ‘endless space or void’, thus using both zero and infinity to describe the same entity. (Vanadeep, 2012)

The concepts of zero and infinity have fascinated many mathematicians, including the great 20th century Indian mathematician, Ramanujan, who has been brilliantly portrayed by Robert Kanigel in his book ‘The Man Who Knew Infinity’. Ramanujan forayed into the realm of metaphysics and at one point tried to build a theory of reality around zero and infinity. According to Ramanujan, zero represents Absolute Reality (akin to Nagarjuna’s concept of ‘shunyata’, as defined later in the section on ‘Post-Vedic Period’) and Infinity represents the myriad manifestations of that Reality; the product of zero and infinity is not one number, but all numbers, a phenomenon which corresponds to the multiplicity or diversity of creation. Such intertwining of mathematics and metaphysics can be mind-boggling. Yet, it is fair to say that overall, Mathematics and Philosophy have had a symbiotic relationship. (Kanigel, 1991)

INDUS VALLEY CIVILIZATION

Mathematical developments in Ancient India were to a large extent impelled by religious needs and activities; however, the study of arithmetic and geometry was also motivated by practical considerations, as revealed by remnants of the Indus Valley Civilization, which existed in India around 3000 BC to 1500 BC. Excavations at Harappa and Mohenjo-Daro, two prominent
centers of the Indus Valley Civilization, provide archeological evidence of well-planned towns with proper drainage system, public bath, brick buildings, and wide roads. Thick walls surrounded the Indus Valley cities. Many people lived in sturdy brick houses that had as many as three floors. Some houses had bathrooms and toilets that were connected to a sewer system. The Indus River’s silt provided the Indus Valley civilization with rich topsoil for farming. An irrigation system of canals provided a reliable source of water for growing wheat and barley. (Dowling, 2018)

The Indus Valley excavations provide evidence of the use of basic mathematics with a practical intent, involving weights and measuring scales. A decimal system was already being used during the Indus Valley Civilization, as indicated by Harappan weights corresponding to ratios of 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, and 500, and measuring scales with decimal divisions. The Harappan weights and measures were remarkably precise. A bronze rod marked in units of 0.367 inch (approx. 1 cm) indicates the degree of precision employed in implementing town planning rules, requiring roads of fixed widths to be perpendicular to each other, and homes and drains to be constructed according to specifications. The graded system of precisely marked weights that existed during the Indus Valley Civilization was in all likelihood used for facilitating commerce. (Sykorova, 2006; O’Connor and Robertson, 2000)

The highly sophisticated nature of the Indus Valley civilization is further revealed by excavations that have unearthed evidence of musical instruments, toys and games, pottery, combs, soaps, and medicine. The Indus Valley cities traded with distant foreign cultures, as evidenced by jewelry made in Harappa that has been found as far away as Mesopotamia. Thousands of clay tablets indicate that the people of the Indus Valley developed a writing system that may be even older than Sumerian writing. This great civilization went into decline by about 1700 BC and seems to have been abandoned by about 1500 BC. (Dowling, 2018; Carr, 2014)

VEDIC PERIOD

The beginning of Indian Mathematics is often traced to the Vedic period, generally assumed to have extended from 1500 BC to 500 BC, although according to more recent sources, it is likely that the Vedic period started much earlier. During the Vedic period the Vedas (Rig, Sam, Yaju, and Atharva Vedas), the oldest scriptures of Hinduism, were composed. The Vedas are often attributed to Aryan invaders from the north who purportedly destroyed the Harappan culture. However, recent evidence suggests that the Aryan invasion is a myth created by a few Eurocentric scholars and that it was desiccation or loss of water that caused the Harappans to abandon their habitat and seek shelter elsewhere. There is no evidence of an Aryan homeland outside of India mentioned anywhere in the Vedas; on the contrary, the Vedas speak of the mighty Sarasvati River and other places indigenous to India. To date, no evidence of a foreign intrusion has been found, neither archaeological, linguistic, cultural nor genetic, whereas there is abundant archaeological evidence including the presence of sacrificial altars that go to show that the Harappans were part of the Vedic fold. This clarifies, to a large extent, the mystery behind the advanced technology used in the Harappan civilization, requiring a sophisticated knowledge of mathematics, especially geometry. Elaborate structures like the Great Bath of Mohenjo-Daro, the Lothal harbor or the citadel at Harappa could not have been built without a substantial knowledge of geometry. History books erroneously tell us that Indians gained their knowledge of Geometry from the Greeks. The absurdity of this notion becomes evident when you consider that Harappans had to have the necessary technical knowledge at least 2,000 years before the Greeks. (Pearce, 2002; Rajaram, 2014; Osborn, 2014).
Vedic literature includes mathematical texts known as the ‘Sulba-Sutras’ which contain detailed instructions for the building of sacrificial altars. Harappan sites contain many such altars, thus providing a link between Vedic literature and Harappan archaeology. This scenario further belies the rather preposterous theory that Vedic literature was brought in by a bunch of nomadic invaders; the instructions contained in Vedic literature were needed for building the altars in Harappa. The ‘Sulba-Sutras’ are the oldest mathematical texts known to us. A careful comparison of the ‘Sulba-Sutras’ with the mathematics of Egypt and old Babylonia led the American mathematician, Abraham Seidenberg, to conclude: "... the elements of ancient geometry found in Egypt and old Babylonia stem from a ritual system of the kind found in the ‘Sulba-Sutras’." (Rajaram, 2014; Seidenberg, 1978; Dutta, 2016; Joseph, 1990)

Records of mathematical applications during the Vedic period are mostly found in Vedic texts associated with religion and ritual. The great engineering feats of the Harappans can be viewed as secular spin-offs of the religiously oriented mathematics found in Vedic literature. Mathematical applications also emerged in the agricultural sector. The system of land grants and agricultural tax assessments required accurate measurement of cultivated areas. Individual farmers in a village often had their holdings broken up into several parcels. Since these parcels could not all be of the same shape, local administrators were required to convert rectangular plots or triangular plots to squares of equivalent sizes. Tax assessments were based on fixed proportions of annual or seasonal crop incomes, but could be adjusted upwards or downwards depending on a variety of factors. This meant that revenue administrators had to have adequate knowledge of geometry and arithmetic. (Kapoor, 2014; Osborn and O’Hara, 2009; Joseph, 1990)

‘Mantras’ or religious chants from the early Vedic period (pre-1000 BC) invoke powers of ten from 10**0 all the way up to 10**19, with Sanskrit names for these exponents, such as eka for 1 or 10**0, dasa for ten or 10**1, satam for hundred or 10**2, sahasram for thousand or 10**3, and lokam for 10**19. Mathematics of the Vedic period dealt with addition, subtraction, multiplication, division, fractions, squares, cubes, and roots, all of which are defined in the ‘Narad Vishnu Purana’ attributed to Veda Vyas (pre-1000 BC) (Bhangale, 2013; Kapoor, 2008; Agarwal and Sen, 2014; Mastin, 2010)

Much of the mathematics embedded in the Vedas is found in texts called ‘Vedangas’, ‘Samhitas’, and ‘Brahmanas’. ‘Vedanga Jyotisya’, dealing with astronomy, gives surprisingly accurate values for astronomical measures such as relative sizes of planets, distance of the earth from the sun, length of the day, and length of the year. ‘Vedanga Kalpa’ dealing with rituals includes handbooks on geometry known as ‘Sulba-Sutras’, which state principles of Geometry that were used for the construction of ritual altars. Baudhayana (800 BC) and Apastamba (600 BC) are the authors of two of the more mathematically important ‘Sulba-Sutras’. Although the principles of Geometry are explicitly stated only in the ‘Sulba-Sutras’ of the late Vedic period, they were known and applied during the earlier phases of the Vedic period. This is evident from the detailed descriptions of techniques for constructing Vedic altars found in the older texts such as ‘Taittriya Samhita’ and ‘Satapatha Brahmana’. In fact, the ‘Sulba’ authors emphasize that they are merely stating facts already known to the authors of the ‘Brahmanas’ and ‘Samhitas’. The Vedic texts display an understanding of basic geometrical shapes and techniques for converting one geometric shape, such as a rectangle, to another shape, such as a square, of equivalent area. ‘Satapatha Brahmana’ gives approximations for the value of π,
which seem to have resulted from transformations of squares into circles and circles to squares. (Pearce, 2002; Dutta, 2016)

The well-known Pythagorean theorem states that the square of the hypotenuse of any right-angled triangle equals the sum of squares of the other two sides; thus, \( c^2 = a^2 + b^2 \), where \( c, a, \) and \( b \) are lengths of the hypotenuse and the other two sides, respectively. Numbers that satisfy the condition \( a^2 + b^2 = c^2 \) are called ‘Pythagorean triples’. Pythagoras, the ancient Greek mathematician who lived in the 6th century BC, is usually credited with the Pythagorean theorem. However, as early as the 8th Century BC, long before Pythagoras, Boudhayana’s Sulba-Sutra listed several Pythagorean triples such as (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25), and (12, 35, 37), as well as a statement of the Pythagorean theorem: “The rope stretched along the diagonal of a rectangle produces an area which its vertical and horizontal sides produce together”. Aside from the Pythagorean theorem, the Sulba-Sutras contain several important results, including geometric solutions of linear and quadratic equations. Apasthamba’s Sulba-Sutra, which builds up on Baudhayana’s Sulba-Sutra, provides a remarkably accurate figure for the square root of 2, obtained from 1 + \( \frac{1}{3} + \frac{1}{(3 \times 4)} - \frac{1}{(3 \times 4 \times 34)} \) which yields a value of 1.4142156, correct to 5 decimal places. (Dutta, 2016; Joseph, 2011; O’Connor and Robertson, 2000; Kapoor, 2008)

The scientific method was formalized in the ‘Nyaya-Sutras’ of the Vedic period, indicating that proofs of the Pythagorean theorem and other results must have been available during this period. However, these proofs, if they did exist, must have been lost, destroyed, or transmitted orally through the ‘Gurukul’ system, with only the results stated in the Vedic texts. On the other hand, Western Mathematicians have given a great deal of attention to proofs, as exemplified by Euclid’s rigorous proof of the Pythagorean theorem. However, according to Prof. Bhargava, who is quoted by Prof. Dutta, “Several verses in the Sulba-Sutras make it clear that the Sulba authors knew why the theorem is true”. Prof. Dutta quotes one such verse from Baudhayana Sulba as follows:

“To combine two squares [with sides \( a \) and \( b \)], mark out the rectangle from the larger square with the side \( b \) of the smaller square. The diagonal of this rectangle \( c \) is the side of a square equal to sum of the two squares.”

By completing the square with side \( c \) and observing that the two triangles excluded from this square are replaced by congruent triangles added to this square, it is obvious that the area of the square with side \( c \) is the sum of the areas of the squares with sides \( a \) and \( b \). Thus, the proof of the Pythagoras theorem is implicit in Baudhayana’s method of combining two squares to form a third square. Therefore, the proof per se could not have eluded Baudhayana who was the first to explicitly state the Pythagoras theorem. (Kapoor, 2008; Dutta, 2016; Joseph, 2011; Pearce, 2002; Iqbal, 2007)

Mathematics or ‘Ganit’, as it was called in Sanskrit, was given considerable importance in the Vedic period, as evidenced by the following statement in ‘Vedang Jyotish’ (1000 BC): "Just as
the feathers of a peacock and the jewel-stone of a snake are placed at the highest point of the body (at the forehead), similarly, the position of Ganit is the highest amongst all branches of the Vedas and the Shastras”. The Vedic period also witnessed the pioneering work of the Sanskrit grammarian, Panini (6th century BC), who provided a scientific notational model that spurred the use of abstract notation in characterizing algebraic equations and presenting algebraic results in a scientific format. In a paper entitled *Panini-Backus form*, the author, Ingerman, finds Panini's notation to be equivalent in its power to that of Backus, the inventor of the Backus Normal Form, used in describing the syntax of programming languages. Distinguished linguists such as Staal, Ingalls, Matilal, Briggs, and Kak have stated that many current developments in formal logic, linguistics, and computer science can be viewed as a rehash of the work of ancient Indian grammarians such as Panini. Western linguists did not see the significance of the context-sensitive rules of Panini’s grammar until Panini-style structures were first introduced by western linguists such as Chomsky about thirty years ago. According to the distinguished linguist Frits Staal: “We can now assert, with the power of hindsight, that Indian linguists in the fifth century BC knew and understood more than Western linguists in the nineteenth century AD (Kapoor, 2008; Bhate and Kak, 1993; Joseph, 2011)

Mathematics in the Vedic period was a byproduct of Hinduism. Later, Jainism and Buddhism also had a profound influence on the development of Mathematical concepts. Like the Hindu world view, space and time were considered endless in Jain cosmology. As early as the 3rd or 2nd Century BC, Jain mathematicians recognized five different types of infinities: infinite in one direction, in two directions, in area, infinite everywhere and perpetually infinite. Jain epistemology allowed for a degree of indeterminacy in describing reality, which probably helped Jain mathematicians in dealing with indeterminate numbers in equations and finding numerical approximations to irrational numbers. The Buddhist literature also refers to indeterminate numbers and infinite sets of numbers. The theory of permutations and combinations is developed in the Jain texts of ‘Bhagvati Sutra’ (3rd century BC) and ‘Sathananga Sutra’ (2nd or 3rd century BC). Jain mathematicians also showed the connection between combinatorial mathematics and coefficients occurring in the binomial expansion. The concept of logarithms is developed in the Jain text of ‘Anuyoga Dwara Sutra’ (2nd century BC). (Kapoor, 2008; Pearce, 2002)

POST-VEDIC PERIOD

One of the most prominent post-Vedic scholars in India who contributed to Mathematics was Pingala (3rd century BC), a musical theorist; Pingala authored the ‘Chandah Sutra’, a Sanskrit treatise on prosody. In the process of enumerating syllabic combinations, Pingala discovered the Pascal triangle, a triangular array, which can be used to recursively generate coefficients occurring in the binomial expansion. This is another instance where a mathematical concept is named after a European mathematician (Pascal (1623 -1662)), although the concept was known in India several centuries earlier. Pingala’s work also contains the basic ideas of the Fibonacci sequence, named maatrumeru by Pingala. Pingala’s work has not survived in its entirety; however, a commentary on it by Halayudha, a 10th century mathematician, has withstood the ravages of time. Halayudha refers to the Pascal triangle as Meru-prastara, meaning staircase to Mount Meru (regarded as the abode of the Hindu Gods) and gives a detailed description of the Pascal triangle as follows: “Draw a square. Beginning at half the square, draw two other similar squares below it; below these two, three other squares, and so on. The marking should be started by putting 1 in the first square. Put 1 in each of the two squares of the second line. In the third line put 1 in the two squares at the ends and, in the middle square, the sum of the digits in the two squares lying above it. In the fourth line put 1
in the two squares at the ends. In the middle ones put the sum of the digits in the two squares above each. Proceed in this way. Of these lines, the second gives the combinations with one syllable, the third the combinations with two syllables, etc.” Pascal’s triangle was described by Halayudha 700 years before it was stated by Pascal, and Halayudha was only restating a rule discovered by Pingala more than 1200 years earlier. Halayudha’s commentary also indicates that Pingala was aware of the combinatorial identity: \( nC0 + nC1 + nC2 + \ldots + nCn-1 + nCn = 2^n \). (Joseph, 2011; Shah, 1991; Dutta-2, 2002; Fowler, 1996)

Nagarjuna (c. 150 AD – 250 AD) was a scholar-monk at Nalanda University, the great Buddhist center of learning in India. Nāgārjuna was one of the most original and influential thinkers in the history of Indian philosophy. He postulated a middle position between existence and nonexistence. That middle position is shunyata or ‘emptiness’. Shunyata, which is the true nature of reality, does not mean nonexistence or the absence of existence altogether, but rather, it means the absence of intrinsic existence, namely existence that is independent and permanent. Everything one encounters in life is devoid of absolute identity or permanence, because everything is dependent on something else; everything arises and passes away according to causal conditions. Belief in the extreme position of permanent or intrinsic existence of anything can cause undue suffering when it ceases to exist. Belief in the extreme nihilistic position that nothing whatsoever exists in any form is also disquieting, because then what we do or do not do makes no difference, leaving us in a complete vacuum, with no sense of direction. Instead, being cognizant of ‘shunyata’, the middle or neutral position, and the transient nature of everything else is the first step toward detachment and eventually liberation or ‘nirvana’. As mentioned earlier, zero or ‘shunya’ as it was referred to by ancient Indian mathematicians, could well have been inspired by the concept of ‘shunyata’. Indeed, zero represents the middle or neutral position between positive numbers, quantifying properties or possessions and negative numbers, quantifying debts at the other extreme. Thus, Nagarjuna was perhaps one of the early thinkers who had a role in the genesis of ‘zero’.

Nagarjuna is also credited with developing the following 4x4 magic square:

\[
\begin{array}{cccc}
30 & 16 & 18 & 36 \\
10 & 44 & 22 & 24 \\
32 & 14 & 20 & 34 \\
28 & 26 & 40 & 06 \\
\end{array}
\]

This is a ‘diabolic magic square with not only the numbers in each row and column and the diagonal summing to 100, but also the numbers in each two-by-two square summing to 100. (Aczel, 2015; Loy, 2006, Selin, 1997)

**GOLDEN AGE**

The Golden Age of Indian mathematics is usually considered to be the period from the 5th century AD to the 12th century AD; many of the mathematical discoveries during this period predated similar discoveries in the West by several centuries, suggesting the possibility of plagiarism by later European mathematicians, at least some of whom were probably aware of the earlier Indian work, which was being propagated by the Arabs. (Mastin, 2010)

While the western world was struggling with the cumbersome roman numeral system all the way up to the thirteenth century, and the Chinese were being hampered by their pictorial script, the situation in India was conducive to the development of many important mathematical
concepts, including the concept of zero. There was already a long and established history in the use of decimal numbers. Further, philosophical and cosmological constructs encouraged a creative approach to number theory, and Panini’s pioneering work on a general scientific notational model led to the extensive use of mathematical notation. Trade, which flourished during the Golden Age, contributed to the growth of mathematics during this period. Brahmagupta’s description of negative numbers as debts and positive numbers as properties is indicative of a link between trade and mathematics. Lending and borrowing in the context of trade required an understanding of both simple and compound interest, which probably stimulated an interest in arithmetic and geometric series. Mathematical developments in the Golden Age were to a large extent also linked to corresponding developments in Astronomy. Knowledge of astronomy, especially knowledge of the tides and the stars, was essential to trading communities who crossed oceans or deserts at night. Anyone who wished to embark on a commercial venture had to gain some grounding in astronomy. This led to a proliferation of astronomy teachers, who received training at universities such as those at Kusumpura (Bihar) or Ujjain (Central India). There was also considerable exchange of information on astronomy and mathematics among scholars, resulting in the transmission of knowledge from one part of India to another. The science of Astronomy was also spurred by the need to have accurate calendars and a better understanding of climate and rainfall patterns for agricultural purposes. (Kapoor, 2008; Dutta-2, 2002; O’Connor and Robertson, 2000)

Golden Age mathematicians in India made fundamental advances in Trigonometry. They used trigonometric concepts like the sine, cosine and tangent functions (which relate the angles of a triangle to the relative lengths of its sides) to survey the land around them, navigate the seas, and even chart the heavens. For instance, Indian astronomers used trigonometry to calculate the relative distances between the Earth and the Moon and the Earth and the Sun. A text dating back to around 400 AD, called the “Surya Siddhanta”, whose authorship is unknown, contains the roots of modern trigonometry, including the earliest use of the sine, cosine, inverse sine, tangent and secant functions. Later mathematicians, including the great mathematician and astronomer Aryabhata (476 AD-550 AAD), made references to this text; not too surprisingly, this text was also translated into Arabic. (Mastin, 2010; Pearce, 2002; Joseph, 2011; Iqbal, 2007)

Aryabhata provided a systematic treatment of the position of the planets in space. He correctly posited the axial rotation of the earth, and inferred correctly that the orbits of the planets were ellipses. He also provided a rational explanation for the solar and lunar eclipses, defying the superstitions and mythical beliefs surrounding these phenomena. Aryabhata had to rely heavily on mathematical concepts in order to gain an understanding of the solar system. In Surya Siddhanta, it was stated: “As the Earth is round, every person considers himself at the top of the Earth where he is standing. So, the downward direction is towards the center of the earth for everyone”. In his classic work, ‘Aryabhatiya’, Aryabhata affirmed that the earth is spherical and further stated that the earth rotates around the Sun. It was 1000 years later that Nicholas Copernicus, who is generally credited with the ‘Helio-centric theory’, proposed this theory, opposing Ptolemy’s ‘Geo-centric theory’. (Vanadeep, 2012; Kapoor, 2008; O’Connor and Robertson, 2000; Dutta, 2006; New World Encyclopedia, 2016)

In the course of determining the precise time of the occurrence of a lunar eclipse, Aryabhata introduced the concept of instantaneous motion or ‘tatkalika gati’ of the moon, which corresponds to the notion of a derivative in modern day calculus; he expressed it in the form of
a basic differential equation, thus sowing the seeds of differential calculus. Aryabhata’s work
was elaborated on by Manjula (10th Century AD) and Bhaskara II (12th Century AD) who
derived the differential of the sine function. Bhaskara II and later mathematicians including
Narayana Pandita (c. 1350) and Jyestadeva (c. 1550) used the method of exhaustion in deriving
the areas of curved surfaces and the volumes enclosed by them, thus introducing the concept
of integration. (Kapoor, 2008; Joseph, 2011)

Aryabhata knew that the ratio of the circumference of a circle to its diameter is a constant,
which we now call ‘pi’, and arrived at an approximate value of 3.1416 (correct up to 4 decimal
places) for this constant, as seen from the following passage from Aryabhata’s work,
‘Aryabhatiya’; “Add 4 to 100, multiply by 8 and add to 62,000. This approximately (aasanna)
is the circumference of the circle, whose diameter is 20,000.” This means that a circle with
diameter 20,000 units has circumference approximately equal to 62832 units, implying that pi = 3.1416, approximately. Aryabhata used this value of pi to estimate the circumference of the
Earth, arriving at a figure of 24,835 miles, only 70 miles off the true value. Further, as suggested
by the use of the word aasanna (approximate) by Aryabhata, perhaps he was aware that pi is
an irrational number, and that any calculation can only be an approximation; many centuries
later, in 1761, the European mathematician, Lambert, proved that pi is indeed an irrational
number. Aryabhata also calculated the length of the solar year with remarkable accuracy
(within 13 minutes of the modern calculation). In making such calculations, Aryabhata had to
solve several problems in algebra (beej-ganit) and trigonometry (trikonamiti). ‘Aryabhatiya’,
containing many of Aryabhata’s results, was translated into Arabic by Abu'l Hassan al-Ahwazi
(before 1000 AD) as ‘Zij al-Arjabhar’ and it is partly through this translation that Indian
computational and mathematical methods were introduced to the Arabs, and eventually to the
Europeans via the Arabs. Incidentally, it is conceivable that the etymology of ‘trigonometry’
is linked to the strikingly similar Sanskrit word, ‘trikonamiti’ (meaning measurement of
triangles), used by ancient Indian mathematicians. (Pearce, 2002; O’Connor and Robertson,

Aryabhata made several other contributions to mathematics. What follows is a list of these
contributions, which is necessarily brief and incomplete, although it is representative of the
broad spectrum of mathematical results we owe to Aryabhata. Aryabhata gave formal
definitions of sine, cosine, versine and inverse sine functions, and provided complete sine and
versine tables, in 3.75° intervals from 0° to 90°, which were accurate up to 4 decimal places;
derived the trigonometric result: \(\sin (n + 1)x - \sin nx = \sin nx - \sin (n - 1)x - (1/225)\sin nx\);
provided several results in spherical trigonometry, which were required for calculations in
astronomy; provided continuous fraction representations of real numbers, including irrational
numbers; and provided solutions to simultaneous quadratic equations as well as solutions to
simultaneous linear equations, including Diophantine equations. (O’Connor and Robertson,

The mathematical tradition sparked by Aryabhata was continued by Varahamihira (505 AD –
587 AD), an eminent astronomer/mathematician. Varahamihira compiled previously written
texts on astronomy into a single text, called the ‘Pancha Siddhanta’ (Five Astronomical
Principles). He also wrote ‘Brihat Samhita’, an encyclopedic work on architecture, temples,
planetary motions, eclipses, rainfall, agriculture, mathematics, and many other topics. Some of
his work has been translated into Arabic or Persian by Al-Biruni, the noted 11th century Persian
scholar. Varahamihira made important additions to Aryabhata’s trigonometric formulas. He
derived the fundamental results relating sine and cosine functions, and provided sine and cosine tables as well. Further, he worked on permutations and combinations, supplementing the results previously derived by Jain mathematicians. He also formalized the method of calculating nCr (number of combinations of n distinct objects taken r at a time) using the triangular array (now known as Pascal’s triangle), discovered earlier by Pingala (6th century BC), and rediscovered much later by the French mathematician, Pascal (1623 – 1662). (Pearce, 2002; O’Connor and Robertson, 2000; Kapoor, 2008; Vijna Bharati, 2018)

Brahmagupta (598 AD-668 AD), the great Mathematician/Astronomer whose path-breaking contributions have already been discussed, included two chapters, 12 and 18, on arithmetic and algebra, respectively, in his book ‘Brahmasphutasiddhanta’. Chapter 12, containing 66 Sanskrit verses, was divided into two sections: "basic operations" (including cube roots, fractions, ratio and proportion, and barter) and "practical mathematics" (including mixture, mathematical series, plane figures, stacking of bricks, sawing of timber, and piling of grain). In the section on practical mathematics, Brahmagupta stated his famous theorem on the diagonals of a cyclic quadrilateral, and provided a formula for the area of a cyclic quadrilateral, which amounted to a generalization of Heron’s formula for the area of a triangle. He also dealt with rational triangles or triangles with sides whose lengths are rational numbers. Chapter 18 of ‘Brahmasphutasiddhanta’, containing 103 Sanskrit verses, began with rules for arithmetic operations involving zero and negative numbers, which have already been discussed. Brahmagupta gave an explicit solution to the quadratic equation, ax**2 +bx = c; he also developed a recursive method to find solutions to Pell's equation (x**2 – ny**2 =1, where n is a given positive non-square integer and integer solutions are sought for x and y) and other quadratic indeterminate equations almost a thousand years before John Pell (1611-1685) who was erroneously given credit by Euler and Lagrange. Another result presented by Brahmagupta is his algorithm for computing square roots; this algorithm is equivalent to the Newton-Raphson iterative formula, but clearly pre-dates it by many centuries. Long before Newton discovered ‘gravity’, Brahmagupta had stated, “Bodies fall towards the Earth because of the nature of the Earth to attract matter”, and had used the Sanskrit term "gruhtvākarṣan" for gravity. Brahmagupta’s copious contributions to Astronomy, including a hand book entitled ‘Karana Khandakāhyaka’ have been of great value to Indian and Arab astronomers and scholars for many centuries. (Stillwell, 2004; Hayashi, 2003; Pearce, 2000; O’Connor and Robertson, 2002; Pickover, 2008; Plofker, 2007)

Bhaskara I (600 AD – 680 AD), Brahmagupta’s cotemporary, also made important contributions to mathematics and astronomy. Bhaskara I continued the work of Aryabhata, and discussed in further detail topics such as the longitudes of the planets, conjunctions of the planets with each other and with bright stars, risings and settings of the planets, and the lunar crescent. These investigations in astronomy required even more advanced mathematics. Bhaskara I extended the work of Aryabhata in his books entitled ‘Mahabhaskariya’, ‘Aryabhatiya-bhashya’, and ‘Laghu-bhaskariya’. He provided a rational approximation of the sine function, and a formula for calculating the sine of an acute angle without the use of a table, correct to two decimal places. He also did pioneering work on indeterminate equations and considered for the first time quadrilaterals with all the four sides unequal and none of the opposite sides parallel. (Kapoor, 2008; O’Connor and Robertson, 2000; Joseph, 2011)

Lalla (720 AD - 790 AD) was an Indian astronomer and mathematician who followed the tradition of Aryabhata I. His famous work entitled ‘Shishyadhividdhidatantra’ was written in two volumes. The first volume on ‘the computation of the positions of the planets’ covered topics such as longitudes of the planets, diurnal motion, lunar and solar eclipses, and
conjunctions of the planets with the fixed stars. In the second volume on ‘the sphere’, Lalla examined topics such as graphical representation, the celestial sphere, the principle of mean motion, the terrestrial sphere, motions and stations of the planets, geography, and instruments for observing astronomical phenomena. (O’Connor and Robertson, 2000; Plofker, 2007)

Acharya Virasena (792 AD-853 AD) was a Jain philosopher and mathematician who wrote a commentary on Jain mathematics, entitled ‘Dhavala’; this commentary deals with the concept of ‘ardhaccheda’, the number of times a number could be divided by 2, effectively base 2 logarithms, and extensions to base 3 (trakacheda) and base 4 (chaturchhedha) logarithms. Virasena’s other contributions include the derivation of the volume of a frustum using an iterative procedure, and a more accurate approximation (3.14159292) for pi than the approximation (3.1416) given by Aryabhata. (Joseph, 2011; Gupta, 2000)

Mahavira Acharya (800 AD-870 AD), a Jain mathematician, wrote a book entitled ‘Ganit Saar Sangraha’ where he described a method of calculating the Least Common Multiple of any set of numbers, which is still the accepted method. He also wrote treatises on a wide range of mathematical topics, including permutations and combinations, squares, cubes, square roots, cube roots, plane geometry, solid geometry, the area of an ellipse, the area of a quadrilateral inscribed within a circle, the sum of a series whose terms are squares of terms in an arithmetic progression, and determinate as well as indeterminate polynomial equations. (Kapoor, 2008; Joseph, 2000; Selin, 1997)

Sridhara (870 AD-930 AD), a mathematician from the Bengal region of India, wrote three books entitled ‘Nav Shatika’, ‘Tri Shatika’, and ‘Pati Ganita’. In these books, Sridhara provided mathematical formulae for a variety of practical problems involving ratios, barter, simple and compound interest, mixtures, purchase and sale, rates of travel, wages, and filling of cisterns. He also dealt with rules for calculating the volume of a sphere, solving quadratic equations, extracting square and cube roots, and summing terms of various arithmetic and geometric series. (Kapoor, 2008; Joseph, 2000; O’Connor and Robertson, 2000; Shukla, 1959)

Manjula (10th century AD) provided an elegant solution to Aryabhata’s differential equation (mentioned earlier), pertaining to the time of occurrence of a lunar eclipse, by substituting an approximate expression for one of the terms in the differential equation. (Joseph, 2011)

Aryabhata II (920 AD–1000 AD) is the author of the astronomical treatise, ‘Mahasiddhanta’. This treatise consists of 18 chapters; the first 12 chapters deal with mathematical astronomy; the remaining six chapters form a separate section called the Goladhiya (lesson on the sphere) where topics in geometry, geography, and algebra are discussed. Aryabhata II gave detailed rules to solve the indeterminate equation: \( by = ax + c \), a method to calculate the cube root of a number, and a sine table correct up to five decimal places. (O’Connor and Robertson, 2000; Vagiswari, 2007)

Sripati Mishra (1019 AD–1066 AD) wrote ‘Siddhanta Shekhara’, ‘Dhikotidakarana’, and ‘Dhruvamanasa’, major works on astronomy, dealing with solar and lunar eclipses, planetary transits, and planetary longitudes. He also wrote ‘Ganit Tilaka’, an incomplete arithmetical treatise, focused on solutions of quadratic equations and simultaneous indeterminate linear equations. (O’Connor and Robertson, 2000; Sinha, 1985)

Bhaskara II (1114 AD-1185 AD) was one of the most accomplished of all the great Indian mathematicians and an eminent astronomer as well. He was born in Bijapur, located in the state
of Karnataka in South India. He was the Head of the astronomical observatory at Ujjain, the leading mathematical center of ancient India. Bhaskara II is credited with explaining the previously misunderstood operation of division by zero and providing an elegant definition of the concept of infinity, as mentioned earlier. He noticed that dividing one into smaller and smaller pieces yields larger and larger number of pieces. Ultimately, therefore, dividing one into pieces of zero size would yield infinitely many pieces, indicating that $1 \div 0 = \infty$ (symbolizing infinity). Bhaskara II also made important contributions to many different areas of mathematics from solutions of quadratic, cubic and quartic equations (including negative and irrational solutions) to solutions of Diophantine equations of the second order to concepts of infinitesimal calculus and mathematical analysis to spherical trigonometry and other aspects of trigonometry. Some of his findings clearly predate similar discoveries in Europe by several centuries. In particular, his work on calculus predates that of Newton and Leibniz by over half a millennium. Bhaskara II used the ‘chakravala’ method to provide a solution to the general form of Pell’s equation and the general indeterminate quadratic equation; stated Rolle’s theorem, a special case of the mean value theorem, which is one of the most important theorems of calculus; derived the derivative of the sine function; and calculated the length of the Earth's revolution around the Sun as 365.2588 days, accurate up to 2 decimal places. He also derived the well-known trigonometric identities:

\[
\sin(a+b) = \sin(a) \cos(b) + \sin(b) \cos(a)
\]

and

\[
\sin(a-b) = \sin(a) \cos(b) - \sin(b) \cos(a)
\]

(Mastin, 2010; Pearce, 2002; O’Connor and Robertson, 2000; New World Encyclopedia, 2016; Joseph, 2011)

Bhaskara II wrote his magnum opus, ‘Siddhantasiromani’, in 1150 AD at the age of 36. This work, consisting of about 1450 verses, is divided into four parts: ‘Lilavati’ dealing with arithmetic has 278 verses; ‘Bijaganit’, dealing with Algebra has 213 verses; ‘Goladhya’ dealing with the celestial globe has 451 verses; and ‘Grahaganit’, dealing with mathematics of the planets has 501 verses. He also wrote the treatise ‘Karanakutuhala’, focusing on astronomical observations of planetary positions, conjunctions, eclipses, and cosmography. These treatises were later transmitted to the Middle East and eventually to Europe. ‘Lilavati’ was translated into Persian by Abul Faizi in the court of Akbar in 1587 AD. ‘Beeja Ganitam’ was translated into Persian by Attah-ullah-Rushdie in 1634 AD. (Vanadeep, 2012; Satyanarayana, 2015; Encyclopaedia Britannica, 2019)

Bhaskara II not only wrote scholarly treatises, but was also adept at presenting mathematics in a lucid and entertaining manner for popular consumption and to help students and beginners understand mathematical concepts. Several problems in the category of recreational mathematics are found in ‘Lilavati’, which was written for his daughter, Lilavati. These problems are addressed to Lilavati who was probably a very bright young woman. Two of these problems are cited below as examples of the amazing creativity with which Bhaskara II wrote ‘Lilavati’.

Example 1 from ‘Lilavati’: One-fifth of a swarm of bees flew towards a lotus flower, and one-third towards a banana tree. A number equal to three times the difference between the two preceding numbers, oh my beauty with the eyes of a gazelle, flew towards a ‘Codaga’ tree. Finally, one other bee, undecided, flew hither and tither, equally attracted by the delightful fragrance of jasmine and pandanus. Tell me, oh charming one, how many bees were there? The answer is 15, which can be derived by letting $x$ equal the number of bees, expressing each of the other unknowns in terms of $x$, and solving for $x$.

Example 2 from ‘Lilavati’: Whilst making love a necklace broke; a row of pearls mislaid; one sixth fell to the floor; one fifth upon the bed; the young woman saved one third of them; one
tenth was caught by her lover; if six pearls remained upon the string; how many pearls were there altogether? The answer is 30, which can be derived by letting x equal the number of pearls, expressing each of the other unknowns in terms of x, and solving for x. As these examples demonstrate, mathematics can indeed be made quite colorful and exhilarating. Perhaps, we need to follow Bhaskara II’s style in teaching mathematics in order to change the common perception of mathematics as an insipid subject that most students study only because it is a mandatory part of their curriculum. (Colebrooke, 1817; Nowlan, 2017)

During the Golden Age of Indian Mathematics, Indian mathematical texts were increasingly being translated into Arabic and Persian. Scholars such as Ibn Tariq and Al-Fazari (8th C), Al-Kindi (9th C), Al-Khwarizmi (9th C), Al-Qayarawani (9th C, author of Kitab fi al-hisab al-hindi), Al-Uqlidisi (10th C, author of ‘The book of Chapters in Indian Arithmetic’), Ibn al-Samh (11th C), Al-Nasawi (11th C.), Al-Beruni (11th C), and Ibn-Al-Saffar (11th C) were among the many who based their own scientific texts on translations of Indian treatises. The contributions of Indian mathematics were generously acknowledged by several important Arabic and Persian scholars, especially in Spain. Eventually, Indian mathematics did reach Europe through a series of translations, but did not receive due acknowledgement by western scholars. (Kapoor, 2008)

KERALA SCHOOL

The following news item appeared in Telegraph, U.K. on 14/08/2007: “A little known school of scholars on the coast of southwest India discovered one of the founding principles of modern mathematics hundreds of years before Newton published them, according to a new study. Dr. George Gheverghese Joseph from the University of Manchester, who did the new study with Dennis Almeida at the University of Exeter, says the 'Kerala School' identified the 'infinite series'—one of the basic components of calculus — some 250 years earlier”. The contributions of the Kerala School have not been widely acknowledged, primarily because of Eurocentricity stemming from European colonialism and the consequent lack of recognition for scientific ideas emanating from the Non-European world. (University of Manchester, 2007)

Some Euro-centric scholars have succeeded in propagating the myth that Indian Mathematics was at a standstill after Bhaskara II until the British introduced modern Mathematics. This cannot be further from the truth. Admittedly, the progress was slowed down in North India; however, in Kerala, this period marked a high point in the development of Astronomy and Mathematics. The remarkable discoveries of Kerala mathematicians include a formula for the ecliptic, the Newton-Gauss interpolation formula, formulas for sums of infinite series, concepts of differentiation and integration, and iterative methods for the solution of non-linear equations. ‘Yuktibhasa’, written by Jyesthadeva (16th century) of the Kerala School explores concepts and methods of calculus, which were rediscovered in Europe much later in the 17th century. Charles Whish, who published the famous paper “On the Hindú Quadrature of the Circle, and the Infinite Series of the Proportion of the Circumference to the Diameter Exhibited in the Four Sástras, the Tantra Sangraham, Yucti Bháshá, Carana Padhati, and Sadratnamála” in the ‘Transactions of the Royal Asiatic Society of Great Britain and Ireland’ in 1834, was one of the first Westerners to recognize that the Kerala school had anticipated by almost 300 years many European developments in Mathematics. (Kapoor, 2008; O’Connor and Robertson, 2000; Katz, 1995; Thrivikraman, 1985; Whish, 1834; Webb, 2014)

Madhava of Sangamagrama (c. 1340 AD – 1425 AD), often called the greatest mathematician-astronomer of medieval India, founded the Kerala School of Astronomy and Mathematics in
the 14th Century. Many of his writings have been lost. However, his contributions to mathematics are highlighted in the ‘Tantasamgraha’, a major treatise written by Nilakantha who came 100 years later. Madhava was the first to use infinite series approximations for a range of trigonometric functions, progressing from the traditional treatment of finite algebraic processes to the consideration of their limit passage to infinity, which is the very basis of modern mathematical analysis. Madhava derived the Taylor-Newton infinite series for the sine and cosine functions and the Gregory series for the inverse tangent function around 1400 AD, over two hundred years before they were rediscovered in Europe. His other contributions include the derivation of an infinite series expression for π, commonly attributed to Leibniz; manipulation of the error term involved in the finite sum for π to obtain a better approximation for π (correct up to 9 decimal places); and the construction of sine and cosine tables, accurate up to 9 decimal places. (Pearce, 2002; O’Connor and Robertson, 2002; Joseph, 2011; Webb, 2014)

Narayana Pandit (c. 1340-1400), one of the earliest of the Kerala mathematicians, authored two documents, an arithmetical treatise called ‘Ganita Kaumudi’ and an algebraic treatise called ‘Bijaganita Vatamsa’. He also wrote an elaborate commentary on Bhaskara II’s ‘Lilavati’, entitled ‘Karmapradipika’. Narayana Pandit’s other contributions include a rule to calculate approximate values of square roots; solutions of indeterminate higher-order equations; derivation of magic squares; and results pertaining to cyclic quadrilaterals. (Pearce, 2002; Joseph, 2011)

Parameshvara (c. 1370–1460) wrote commentaries on the works of Bhaskara I, Aryabhata and Bhaskara II, including ‘Lilavati Bhasya’, a commentary on Bhaskara II’s ‘Lilavati’. His contributions include an outstanding version of the mean value theorem, a mean value type formula for inverse interpolation of the sine function, an iterative technique for calculating the sine of a given angle, a more efficient approximation that works using a two-point iterative algorithm, which is essentially the same as the modern secant method, and an expression for the radius of a circle inscribed in a cyclic quadrilateral, typically attributed to Lhuilier (1782). (Pearce, 2002; O’Connor and Robertson, 2000; Gupta, 1977; Plofker, 1996)

Nilakantha Somayaji (1444–1544) wrote the ‘Tantra Samgraha’ (which 'spawned' a later anonymous commentary ‘Tantrasangraha-vyakhya’ and a further commentary by the name Yuktidipaika, written in 1501). This work can be viewed as an elaboration and an extension of Madhava’s work. Nilakanta Somayaji also authored ‘Aryabhatiya-bhasa’, a commentary on the ‘Aryabhatiya’. His contributions include the concept of mathematical proof by induction, proof of the Madhava-Gregory series for the arctangent, improvements and proofs of other infinite series expansions by Madhava, and the relationship between the power series representation of π/4 and that of arctangent. (Pearce, 2002; O’Connor and Robertson, 2000; Roy, 1990)

Chitrabhanu (c. 1530) was a 16th-century mathematician from Kerala who gave integer solutions to 21 types of systems of two simultaneous algebraic equations in two unknowns. For each case, Chitrabhanu gave an explanation and justification of his rule as well as an example. Some of his explanations were algebraic, while others were geometric. (Joseph, 2009; Hayashi, 1998)

Jyesthadeva (c. 1500–1575) was another notable member of the Kerala School. His magnum opus, 'Yuktibhāṣā' (written in Malayalam, the regional language of Kerala) occupies a special place in the history of mathematics as a seminal text on the basics of calculus, predating related
work of European mathematicians such as Gregory, Wallis, Taylor, Newton and Leibniz by several centuries. In this remarkable work, Jyesthadeva presented proofs of most mathematical theorems and infinite series discovered earlier by Madhava and other Kerala School mathematicians. More specifically, this work contains derivations of the series expansions for the arctangent function (equivalent to Gregory’s proof, which came much later) and the sine and cosine functions. These derivations involved concepts of differentiation and integration as well as indefinite integrals of powers. ‘Yuktibhasa’ also contains derivations of the surface area and volume of a sphere by integration of infinitesimal elements. (Joseph, 2009; Pearce, 2000; Padmanabhan, 2012)

MODERN ERA

The rich tradition of Mathematics handed down to us by ancient Indian mathematicians starting from the Indus Valley era has naturally inspired many Indian mathematicians to continue this tradition through many centuries leading up to the modern era. Thus, we are now able to name quite a few distinguished modern Indian mathematicians. In this galaxy of modern Indian mathematicians, Srinivas Ramanujan, mentioned earlier in the context of ‘Infinity’, clearly stands out. A recent movie entitled ‘The Man Who Knew Infinity’, based on the book with the same title, has shed light on many facets of this outstanding mathematician, which are outlined below.

Srinivas Ramanujan (1887-1920) was born in a modest South Indian family in Tamil Nadu. Ramanujan was a promising student, winning academic prizes in high school. At the age of 16, his life took a decisive turn after he obtained a book entitled ‘A Synopsis of Elementary Results in Pure and Applied Mathematics’. This book was simply a compilation of thousands of mathematical results, written as an aid in coaching English mathematics students facing the notoriously difficult Tripos examination. But this book inspired a burst of feverish mathematical activity in Ramanujan, as he worked through the book's results and beyond. Unfortunately, his total immersion in mathematics, at the expense of all other subjects, was disastrous for Ramanujan's academic career; he repeatedly failed his college exams.

As a college dropout from a rather poor family, Ramanujan's position was precarious. He lived off the charity of friends, filling notebooks with mathematical discoveries and seeking patrons to support his work. Finally, he met with modest success when the Indian mathematician Ramachandra Rao provided him with first a modest subsidy, and later a clerkship at the Madras Port Trust. During this period Ramanujan had his first paper published, a 17-page work on Bernoulli numbers that appeared in 1911 in the ‘Journal of the Indian Mathematical Society’. Still no one was quite sure if Ramanujan was a real genius or a crank. With the encouragement of friends, he wrote to mathematicians in Cambridge seeking validation of his work. Twice he wrote with no response; on the third try, he found Hardy.

It is one of the most romantic stories in the history of mathematics: in 1913, the English mathematician, G. H. Hardy, received a strange letter from an unknown clerk in Madras, India. The ten-page letter contained about 120 statements of theorems on infinite series, improper integrals, continued fractions, and number theory. Every prominent mathematician gets letters from cranks, and at first glance Hardy no doubt put this letter in that category. But something about the formulas made him take a second look, and show it to his collaborator, J. E. Littlewood. After a few hours, they concluded that the results "must be true because, if they were not true, no one would have had the imagination to invent them".
Hardy wrote enthusiastically back to Ramanujan, and Hardy's stamp of approval improved Ramanujan's status almost immediately. Ramanujan was named a research scholar at the University of Madras, receiving double his clerk's salary and required only to submit quarterly reports on his work. But Hardy was determined that Ramanujan be brought to England. Ramanujan's mother resisted at first, but finally gave in, ostensibly after a vision. In March 1914, Ramanujan boarded a steamer for England.

Ramanujan's arrival at Cambridge was the beginning of a very successful five-year collaboration with Hardy. In some ways the two made an odd pair: Hardy was a great exponent of rigor in analysis, while Ramanujan's results were (as Hardy put it) "arrived at by a process of mingled argument, intuition, and induction, of which he was entirely unable to give any coherent account". Hardy did his best to fill in the gaps in Ramanujan's education without discouraging him. He was amazed by Ramanujan's uncanny formal intuition in manipulating infinite series, continued fractions, and the like: "I have never met his equal, and can compare him only with Euler or Jacobi."

Ramanujan's years in England were mathematically productive, and he gained the recognition he hoped for. Cambridge granted him a Bachelor of Science degree "by research" in 1916, and he was elected a Fellow of the Royal Society (the first Indian to be so honored) in 1918. But the alien climate and culture took a toll on his health. Ramanujan had always lived in a tropical climate and had his mother (and later his wife) to cook for him: now he faced the English winter, and he had to do all his own cooking to adhere to his strict vegetarian diet. Wartime shortages only made things worse. In 1917 he was hospitalized, his doctors fearing for his life. By late 1918 his health had improved; he returned to India in 1919. But his health failed again, and he died the next year at the young age of 32, making a permanent exit from the mathematical stage on which he had performed brilliantly. But, his legacy lives on.

Besides his published work, Ramanujan left behind several notebooks, which have been the object of much study. The English mathematician, G. N. Watson, wrote a long series of papers about them. More recently the American mathematician, Bruce C. Berndt, has written a multi-volume study of the notebooks. In 1997 'The Ramanujan Journal' was launched to publish work ‘in areas of mathematics influenced by Ramanujan’.

Ramanujan’s colorful and productive life was filled with divine inspiration. He was an ardent devotee of his family goddess, Mahalakshmi, to whom he attributed his mathematical prowess and accomplishments. He once famously said, “An equation for me has no meaning, unless it represents a thought of God.”

There are numerous other notable Indian mathematicians and computer scientists of the modern era who have made significant contributions. Their publications coupled with the glut of information available on the internet provide readers with easy access to their contributions, thus obviating the need to include them in this article designed primarily to shed light on the less known contributions of ancient Indian mathematicians.

CONCLUSION

Starting with ‘zero’ and the decimal place value system, Indian contributions to mathematics have not been given due recognition as a result of Eurocentrism, which was being stoked by European colonization in many parts of the world. The colonial agenda of the British empire in India also did not favor crediting Indian mathematicians with their discoveries. Western
scholars in the early 20th century had been influenced by Kaye, an anti-Indian scholar who strove relentlessly to spread the message that the Indian number system including the zero numeral was either European or Arabic in origin. Many mathematical results first derived by ancient Indian mathematicians such as Boudhayana, Brahmagupta, Pingala, Bhaskara I and II, and Aryabhata I and II, have been attributed to western mathematicians such as Pythagoras, Pascal, Fibonacci, Copernicus, Pell, Newton, and Leibniz. The infinite series of calculus for trigonometric functions (rediscovered by Gregory, Taylor, Newton, and Maclaurin in the late 17th century) were described (with proofs) in India, by mathematicians of the Kerala school, remarkably some two centuries earlier. Some scholars have suggested that knowledge of these results might have been transmitted from Kerala to Europe by traders and Jesuit missionaries, in light of the fact that Kerala was in contact with Europe from about 1500 AD, the methodological similarities between Indian and European mathematics, the existence of communication routes, and the chronology of mathematical developments. While the Kerala mathematicians were exploring infinite series to obtain better approximations for quantities such as pi, mathematical activity in Europe was in the doldrums. During this time Europe had just begun to use the decimal system which was being used in India since at least the third century. The decimal value system, which makes calculations much easier compared to the Roman numeral system, undoubtedly gave Indian mathematicians a head start. When mathematical developments are thus put in perspective, it is easy to see why the Eurocentric view is simply untenable.

There are notable exceptions to the Eurocentric view, as exemplified by the American historian, Will Durant, the Australian Indologist, A. L. Basham, and the French Mathematician, Pierre-Simon Laplace. They acknowledge that mathematics today owes a huge debt to the outstanding contributions made by Indian mathematicians over many centuries. Considering the pivotal role Mathematics has played in scientific developments, it is fair to say that India’s contributions in the field of mathematics deserve broader recognition.

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